

## Finite Element Analysis of Ribbed Dome

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### ABSTRACT

Large spans have always fascinated people since unobstructed closed places have been demanded for many purposes. Dome structures are the most preferred type of large spanned structures. For understanding the behaviour of ribbed dome structure, in this paper ribbed spherical dome with rigid joints are considered. Three different spans of domes are considered for analysis. The proposed dome will be modeled and analysis to be done by using software's ANSYS and Staad.Pro for different rise to span ratios for different load cases and results are compared. Failure of dome is generally due to buckling of the structure. It is a sudden failure occurs to the structure when it reaches a critical load, which is the maximum load which a member can support before it becomes unstable. Failure of dome structure is due to buckling of members. In the present study buckling load of ribbed spherical dome is calculated using finite element software ANSYS. A Parametric study is also conducting buckling analysis by changing the rise to span ratio of the dome

**Keywords**—Ribbed Dome, Member Buckling, Buckling Load

### I. INTRODUCTION

Large spans have always fascinated people since unobstructed closed places have been demanded for many purposes. Beginning with the worship places in the early times, sports stadium, assembly halls, exhibition centres, swimming pools, shopping malls and industrial buildings have been the typical examples of structures with large unobstructed areas nowadays. Dome structures are the most preferred type of large spanned structures. Domes have been of a special interest in the sense that they enclose a maximum amount of space with minimum obstructions and impressive beauty is the most fascinating one for the designers since the earliest times. Domes have been constructed from a wide variety of building materials over the centuries: from mud to stone, wood, brick, concrete, metal, glass and plastic. Braced steel dome structures have been widely used all over the world during last three decades New production techniques allowed fore cast iron and wrought iron to be produced both in larger quantities and at relatively low prices during the Industrial Revolution.

Present study is focused on the ribbed dome analysis. Shiro Kato [2] studied about collapse of semi-rigidly jointed reticulated domes with initial geometric imperfections. Ronaldo C. Battista [3], studied about strengthening of a reticulated spherical dome against local instabilities. The strength capacity of a reticulated spherical dome is generally associated with inelastic buckling of its slender members and more often of the partially restrained connections between

members. Wenjiang Kang [4] find out tensegric system can greatly improve the mechanical properties of the single-layer dome system. It decreases the nodal displacement by inversely loading the single-layer steel truss and effectively improves the buckling capacity of the system. Y.Q. Maa [7] studied about Buckling of super ellipsoidal shells under uniform pressure. A. Kaveha, [8] has developed an optimum topology design algorithm method which is based on the hybrid Big Bang\_Big Crunch optimization (HBB\_BC) for the Schwedler and ribbed domes. In his study he find out that even if we increase the number of rings in the dome we can't increase the performance of the dome.

The purpose of this paper is to study the behavior of ribbed dome for different loads at apex are considered. In the study the rings of dome is limited to three [3], which shows good performance against different loading conditions at earlier studies Ribbed dome consists of a number of intersecting "ribs" and "rings". A "rib" is a group of elements that lie along a meridian line and a "ring" is a group of elements that constitute a horizontal polygon. Ribs can be radial trussed or solid. They generally interconnect at the crown and a tension ring at the foundation stiffens the ribs. A ribbed dome will not be structurally stable unless it is designed as rigidly-jointed system, since it does not have diagonal elements.

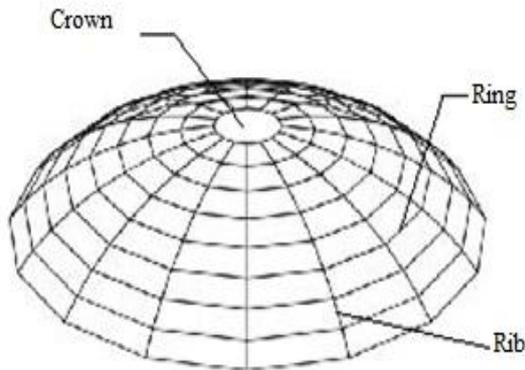


Fig. 1 Typical Ribbed Dome

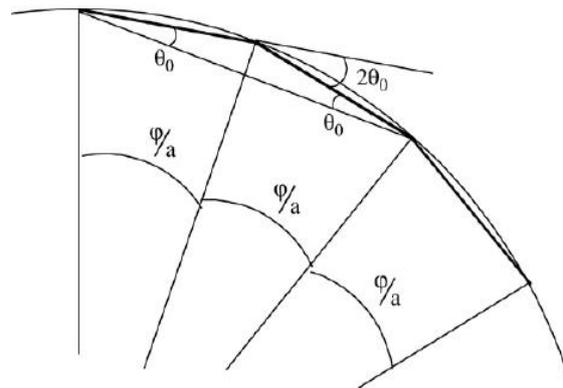


Fig. 3. Meridian section of the dome

## II. GEOMETRIC PARAMETERS OF DOME

In this paper ribbed spherical dome with rigid joints are considered. The spans (D) of the dome considered for analysis are 20m,30m and 40m. Total number of rings in the dome is selected as 3 and it is equally spaced, that is the members in meridian line have same length. The typical geometric scheme of ribbed dome and its member joints are shown in Figs. 2-3. The total angle subtended by the dome can be derived from Fig.2. This angle depends on the rise to span ratio of the dome[6].

$$\tan \varphi = \frac{4 \times D/H}{(D/H)^2 - 4} \quad (1)$$

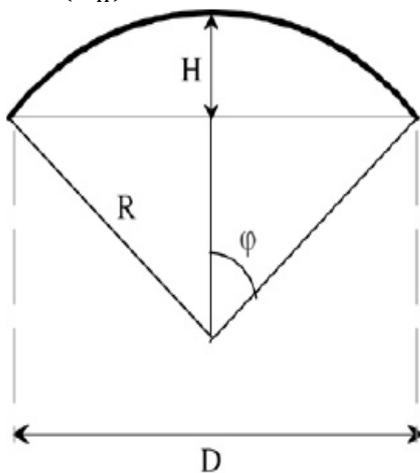


Fig. 2. Scheme for the derivation of Eq. (1)

The total angle is divided equally to determine the position of the rings. The angle between members located along the meridian lines is thus  $2\theta_0$  ( see Fig. 3 ), where

$$\theta_0 = \frac{\varphi}{2a} \quad (2)$$

Where, a = number of rings in dome

This angle has very significant influence on the behaviour of the dome

## III. STRUCTURAL MODEL PARAMETERS

The joints of dome structures are considered to be rigidly connected and the members are exposed to both axial forces and bending moments. Therefore, bending moments of members affect the axial stiffness of these elements because of being slender members. Steel tubes are used for the dome structure. The area (A) and moment of inertia (I) of the section of the members are kept constant for ribs and rings of the dome with different loads acting on the dome. The modulus of elasticity of steel is taken as  $210 \text{ kN/mm}^2$ .

Table . 1 : Member Properties of Domes

| SPAN |      | Size of Member (mm) | Thickness (mm) | Area of Section (mm <sup>2</sup> ) | Moment of Inertia (mm <sup>4</sup> ) X 10 <sup>4</sup> |
|------|------|---------------------|----------------|------------------------------------|--|
| 40m  | Rib  | 200 X 200           | 25             | 17500                              | 9110   |
|      | Ring | 150 X 150           | 25             | 12500                              | 3390   |
| 30m  | Rib  | 160 X 160           | 25             | 13500                              | 4240   |
|      | Ring | 120 X 120           | 25             | 9500                               | 1530   |
| 20m  | Rib  | 110 X 110           | 25             | 8500                               | 1110   |
|      | Ring | 90 X 90             | 20             | 5600                               | 495  |

Different rise to span considered for analysis is in between the values 0.10 to 0.50 with an increment of 0.05. For the study of general behaviour of dome three loading conditions are considered:

Case 1. The vertical downward load of - 500 kN;

Case 2. The two horizontal loads of - 100 kN in the X and Y directions;

Case 3. The vertical downward load of - 500 kN and two horizontal loads of 100 kN in the x and y directions.

And buckling load of the dome is calculated by applying concentrated load at the apex for different rise to span ratio.

### 3.1 Instability Modes of Domes

Failure of dome is generally due to buckling of the structure. It is a sudden failure occurs to the structure when it reaches a critical load, which is the maximum load which a member can support before it becomes unstable.

The buckling of single-layer structures can appear in several ways. In particular, a single-layer dome can exhibit:

(i) *Member buckling*, where the buckling of one member in a single-layer dome can imply the collapse of the structure. Member buckling can be avoided by ensuring an adequate bending stiffness of the members.

(ii) *Node instability*, will occur when the combined axial forces in all of the members attached to a joint cannot balance the external load. When this happens the node experiences a much larger displacement than the neighboring nodes. The dynamic loads involved when the node leaps from one position to a more distant position are very harmful for the whole structure.

(iii) *Line instability*, which appears when all the nodes and members in a ring are involved in the loss of stability

(iv) *General instability*, where the loss of stability simultaneously appears at several nodes.

In this paper buckling analysis of a Ribbed dome using Finite element software ANSYS is discussed for different loads at apex and we are here concentrating on member buckling of the dome since which is normally causes the failure of dome.

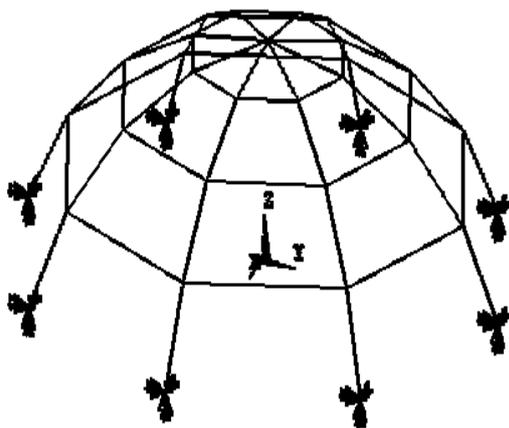


Fig. 4. Isometric View of Ribbed Dome with Fixed Support

### I. FINITE ELEMENT ANALYSIS

For the study of general behaviour of dome three loading conditions are considered at the apex of the dome. Maximum Axial forces in rib & ring members, Maximum moment in the members and Maximum deflection of dome structure are the criteria's chosen for finding out the most effective

rise to span ratio for the given span using ANSYS and compare the results with Staad.Pro. An isometric view of a dome is shown in Fig.4 shown below. The joints of dome structures are considered to be rigidly connected and all the supports are provided as fixed supports.

### II. RESULTS AND DISCUSSION

For the study of general behaviour of dome three loading conditions are considered at the apex of the dome. Maximum Axial forces in rib & ring members, Maximum moment in the members and Maximum deflection of dome structure are the criteria's chosen for finding out the most effective rise to span ratio for the ribbed dome

#### 5.1 Axial Force on Members

Load on domes are mainly transferred to the support through meridian compressive stress and hoop tension in the members that is the arch action of the dome structure. Normally ribs are taking the compressive force and rings are taking the tensile force in the dome when loads are acting.

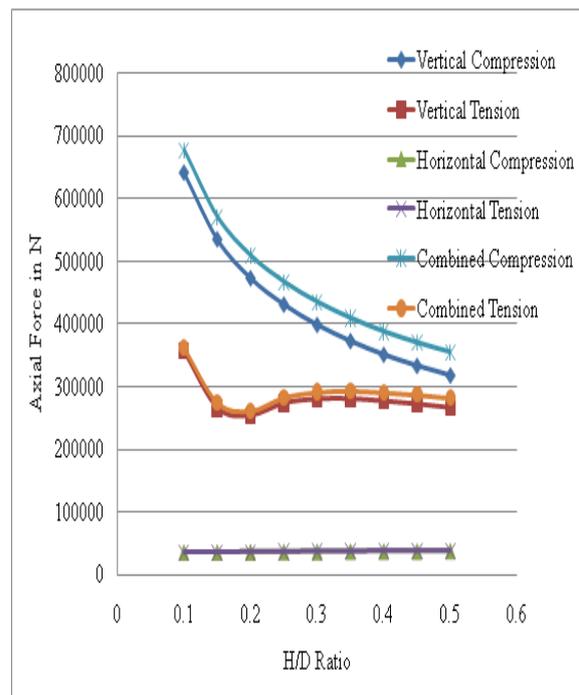


Fig. 5: Axial Force Vs  $H/D$  Ratio for 20m Spanned Dome

Table 2 gives the maximum compressive and tensile forces developed in the dome structures using ANSYS. Fig. 5 gives the graphical representation variation of axial force with different rise to span ratio

Table. 2: Maximum Axial Force on 20m Spanned Dome for Different Load Cases [ANSYS]

| $H/D$ Ratio | Vertical Load   |             | Horizontal Load |             | Combined Loads  |             |
|-------------|-----------------|-------------|-----------------|-------------|-----------------|-------------|
|             | Compression (N) | Tension (N) | Compression (N) | Tension (N) | Compression (N) | Tension (N) |
| 0.10        | 641167          | 355655      | 35574           | 35574       | 676740          | 360676      |
| 0.15        | 534791          | 265391      | 35891           | 35891       | 570682          | 273924      |
| 0.20        | 472927          | 254004      | 36216           | 36216       | 509143          | 260584      |
| 0.25        | 430811          | 273368      | 36525           | 36525       | 467336          | 281930      |
| 0.30        | 398483          | 280062      | 36799           | 36799       | 435282          | 290330      |
| 0.35        | 372594          | 280461      | 37047           | 37047       | 409640          | 292235      |
| 0.40        | 350863          | 276723      | 37262           | 37262       | 388125          | 289748      |
| 0.45        | 333100          | 272041      | 37453           | 37453       | 370553          | 286151      |
| 0.50        | 317688          | 266164      | 37619           | 37619       | 355307          | 281173      |

Table. 3: Maximum Axial Force on 20m Spanned Dome for Different Load Cases [STAAD. Pro]

| $H/D$ Ratio | Vertical Load   |             | Horizontal Load |             | Combined Loads  |             |
|-------------|-----------------|-------------|-----------------|-------------|-----------------|-------------|
|             | Compression (N) | Tension (N) | Compression (N) | Tension (N) | Compression (N) | Tension (N) |
| 0.10        | 624650          | 362634      | 36709           | 36709       | 663438          | 361929      |
| 0.15        | 521014          | 270599      | 37036           | 37036       | 559465          | 274876      |
| 0.20        | 460744          | 258988      | 37371           | 37371       | 499135          | 261489      |
| 0.25        | 419713          | 278732      | 37690           | 37690       | 458150          | 282910      |
| 0.30        | 388218          | 285558      | 37973           | 37973       | 426726          | 291339      |
| 0.35        | 362996          | 285965      | 38229           | 38229       | 401588          | 293250      |
| 0.40        | 341824          | 282153      | 38451           | 38451       | 380496          | 290755      |
| 0.45        | 324519          | 277379      | 38648           | 38648       | 363269          | 287145      |
| 0.50        | 309504          | 271387      | 38819           | 38819       | 348323          | 282150      |

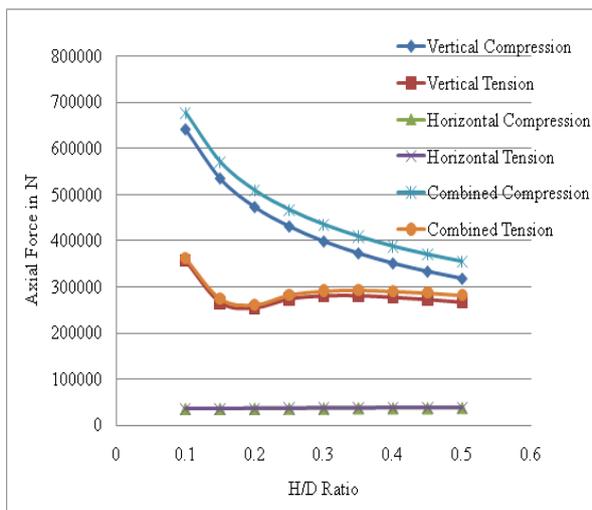


Fig. 6: Axial Force Vs  $H/D$  Ratio for 20m Spanned Dome

Table. 3 gives the maximum compressive and tensile forces developed in the dome structures using Staad Pro. And the Fig. 6 gives the graphical representation variation of axial force with different rise to span ratio of dome structure with Staad Pro.

While comparing the maximum compressive and tensile forces developed in the dome structures using ANSYS and Staad Pro. Values are found out to be almost same and the Fig. 7 gives the graphical representation comparison of axial force with different rise to span ratio for both software's and the difference in values are less than 5%.

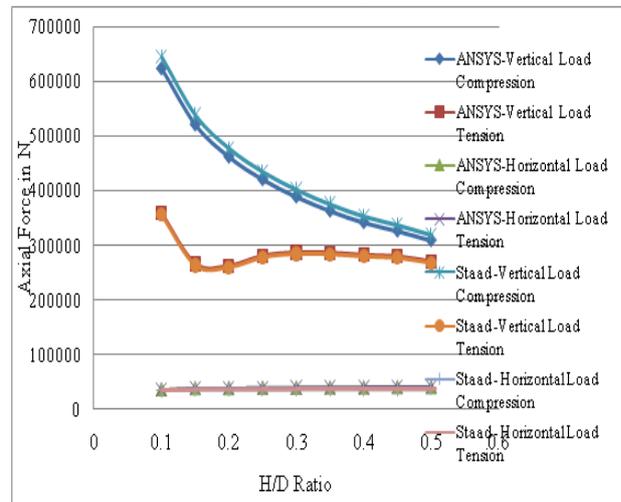


Fig. 7: Comparison of Axial Force for Dome

While studying the axial force in the members of dome it's seen that when vertical load is acting on the dome all the rib members undergo compression and rings undergo tension the system. But when ever horizontal forces acting in the dome apex the ribs in the opposite direction of applied forces and its resultant direction shows tension in the member. It seems that if the  $H/D$  ratio above 0.25 the tension are distributed much more evenly to rings so that, the structure will be more stable. If axial forces on members are considered as deciding factor for selection of rise to span ratio for the ribbed dome structure we can propose a rise to span ratio above 0.25 for circular ribbed dome.

### 5.2 Moment on Members

Due to the rigidity of the joints there will be moments in the dome members. It is not at all feasible to have a large concentration of moment in a member, which will affect the stability of structure. Table 4 gives the Maximum Moment developed in the dome structures using ANSYS. The Fig. 8 gives the graphical representation variation of maximum moment with different rise to span ratio

Table. 4: Maximum Moment on 30m Spanned Dome for Different Load Cases [ANSYS]

| $H/D$ Ratio | Vertical Load X $10^5$ [Nmm] | Horizontal Load X $10^5$ [Nmm] | Combined Loads X $10^5$ [Nmm] |
|-------------|------------------------------|--------------------------------|-------------------------------|
| 0.10        | 210                          | 87.5                           | 297                           |
| 0.15        | 147                          | 107                            | 254                           |
| 0.20        | 109                          | 118                            | 227                           |
| 0.25        | 83.2                         | 126                            | 209                           |
| 0.30        | 65.5                         | 131                            | 196                           |
| 0.35        | 52.5                         | 136                            | 189                           |
| 0.40        | 43                           | 142                            | 185                           |
| 0.45        | 40.9                         | 144                            | 185                           |
| 0.50        | 34.2                         | 150                            | 184                           |

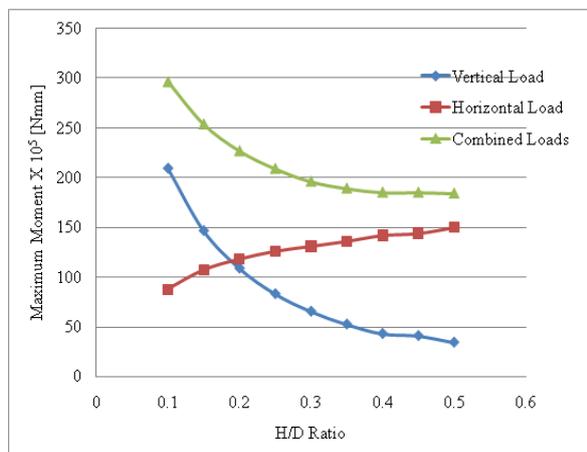


Fig. 8: Maximum Moment Vs  $H/D$  Ratio for 30m Dome

Table 5 gives the Maximum Moment developed in the dome structures using Staad Pro. The Fig. 9 gives the graphical representation variation of maximum moment with different rise to span ratio.

Table. 5: Maximum Moment on 30m Spanned Dome for Different Load Cases [STAAD. Pro]

| $H/D$ Ratio | Vertical Load X $10^5$ [Nmm] | Horizontal Load X $10^5$ [Nmm] | Combined Loads X $10^5$ [Nmm] |
|-------------|------------------------------|--------------------------------|-------------------------------|
| 0.10        | 273.184                      | 106.931                        | 333.914                       |
| 0.15        | 191.229                      | 130.761                        | 285.569                       |
| 0.20        | 141.796                      | 144.204                        | 255.213                       |
| 0.25        | 108.233                      | 153.980                        | 234.976                       |
| 0.30        | 85.207                       | 160.091                        | 220.360                       |
| 0.35        | 68.296                       | 166.201                        | 212.490                       |
| 0.40        | 55.938                       | 173.533                        | 207.993                       |
| 0.45        | 53.206                       | 175.978                        | 207.993                       |
| 0.50        | 44.490                       | 183.310                        | 206.869                       |

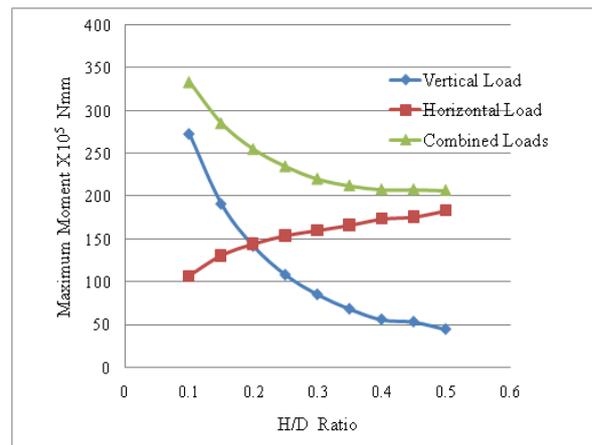


Fig. 9: Maximum Moment Vs  $H/D$  Ratio for 30m Dome

Moment value obtained in staad is higher than the ANSYS values. Maximum moment in a dome is happening at second rib member from the apex of the dome. In the case of horizontal loads in some analysis its seen that maximum negative moment developed at rib member near to support. From the graph upto  $H/D$  ratio 0.30 there is a visible reduction in moments and after that there is not much variation in the moments, so it is better to choose  $H/D$  ratio in between 0.25 to 0.40 for dome if moment on members are considered as deciding factor for selection of rise to span ratio for the ribbed dome structure we can propose a rise to span ratio.

### 5.3 Maximum Deflection of Dome

Deflection of members is the critical factors which need to be checked for the stability of domes. If a joint of a dome shows considerable deflection with respect to other joints in the dome it may lead to joint instability of the dome structure. Since rigid joints are provided chance of local instabilities are less.

Table. 6: Maximum Deflection on 40m Spanned Dome for Different Load Cases [ANSYS]

| $H/D$ Ratio | Vertical Load [mm] | Horizontal Load [mm] | Combined Loads [mm] |
|-------------|--------------------|----------------------|---------------------|
| 0.10        | 40.302             | 21.125               | 40.563              |
| 0.15        | 24.585             | 27.621               | 29.564              |
| 0.20        | 17.869             | 33.095               | 34.288              |
| 0.25        | 14.270             | 38.960               | 39.763              |
| 0.30        | 12.129             | 46.185               | 46.782              |
| 0.35        | 10.734             | 56.730               | 57.192              |
| 0.40        | 9.821              | 70.997               | 71.584              |
| 0.45        | 9.089              | 89.347               | 89.997              |
| 0.50        | 8.81               | 113.215              | 113.936             |

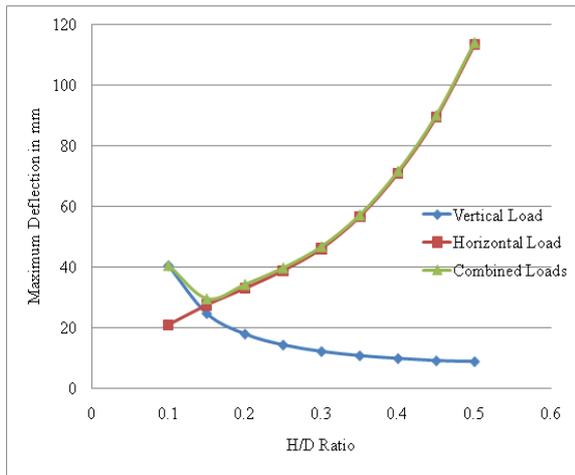


Fig. 10: Maximum Deflection Vs  $H/D$  Ratio for 40m Dome

Table 6 & 7 gives the Maximum deflection occurred in the dome structures using ANSYS and Staad Pro. The Fig. 10 & 11 gives the graphical representation variation of maximum deflection with different rise to span ratio.

Table. 7: Maximum Deflection on 40m Spanned Dome for Different Load Cases [STAAD. Pro]

| $H/D$ Ratio | Vertical Load [mm] | Horizontal Load [mm] | Combined Loads [mm] |
|-------------|--------------------|----------------------|---------------------|
| 0.10        | 37.100             | 23.159               | 44.453              |
| 0.15        | 22.632             | 30.281               | 32.399              |
| 0.20        | 16.449             | 36.282               | 37.576              |
| 0.25        | 13.136             | 42.712               | 43.576              |
| 0.30        | 11.165             | 50.632               | 51.268              |
| 0.35        | 9.881              | 62.193               | 62.676              |
| 0.40        | 9.041              | 77.834               | 78.449              |
| 0.45        | 8.367              | 97.951               | 98.627              |
| 0.50        | 8.110              | 124.117              | 124.862             |

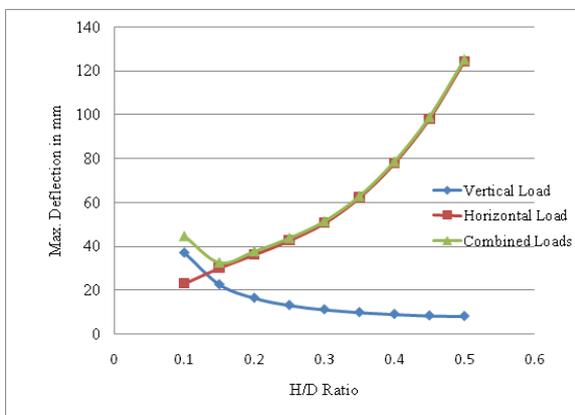


Fig. 11: Maximum Deflection Vs  $H/D$  Ratio for 40m Dome

Ribbed dome shows good performance against the vertical load on the apex. Due to its structural symmetry and shape provide dome good performance against vertical loading. There is not a considerable deflection for the dome structure.

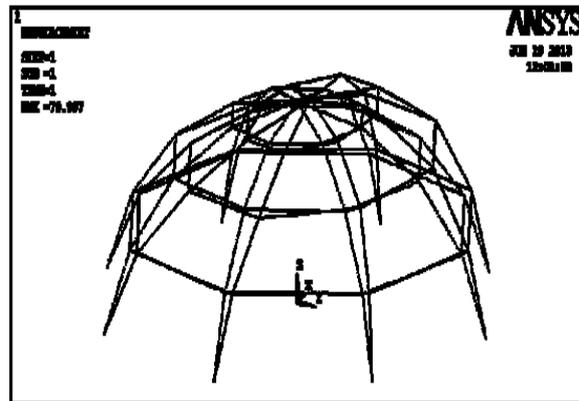


Fig. 12. Deflection due to horizontal force

Since the lateral stiffening for ribbed dome is less, the deflection due to horizontal force is critical for the section. Rib members for the ribbed dome should have much heavier section to resist against lateral loads. There is more stress in the neighbouring elements to supports in dome, while in two other cases the elements near to the apex have the heavier stress. The height of dome should be the minimum for better performance against lateral loading.

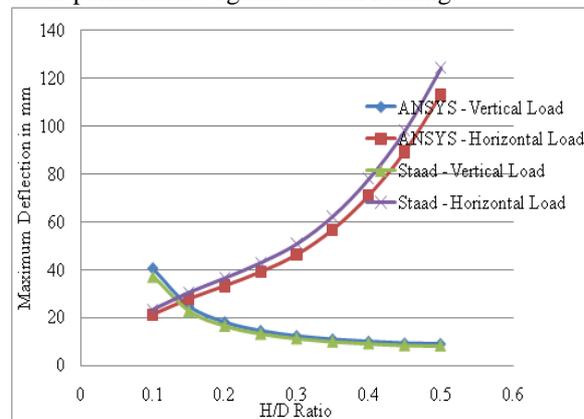


Fig. 13: Comparison of Maximum Deflection in Dome

Deflection of the dome is more due to the presence of lateral load in the system and the stresses the members are high. Ribbed dome shows good performance against the vertical loads. Deflection due to vertical load is slightly lower than the deflection obtained using ANSYS software. But horizontal deflection using Staad is found to be a little higher than the former one. With permissible deflection values we need to choose another  $H/D$  ratio for dome. From the values we can choose  $H/D$  ratio in

between 0.25 to 0.35 as most suitable choice for ribbed dome.

#### 5.4 Effect of Surface Material in Deflection of Dome

Dome structure is actually a shell structure. Effect of surface element in axial force and bending moment of rib and ring member of domes are small. But the effect of plate element in controlling deflection of structure is more due to its membrane action. In ANSYS SHELL63 element is used for the assigning surface material to the dome structure. SHELL63 has both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedom at each node.

Table 8: Maximum Deflection on 20m Spanned Dome for Different Load Cases [ANSYS]

| $H/D$ Raito | Vertical Load [mm] | Horizontal Load[mm] | Combined Loads[mm] |
|-------------|--------------------|---------------------|--------------------|
| 0.10        | 44.087             | 21.624              | 44.395             |
| 0.15        | 25.809             | 25.787              | 27.850             |
| 0.20        | 21.013             | 29.661              | 30.551             |
| 0.25        | 14.574             | 33.949              | 34.458             |
| 0.30        | 12.276             | 39.082              | 39.512             |
| 0.35        | 10.923             | 44.171              | 44.524             |
| 0.40        | 9.983              | 51.100              | 51.458             |
| 0.45        | 9.368              | 59.953              | 60.403             |
| 0.50        | 8.908              | 75.400              | 75.853             |

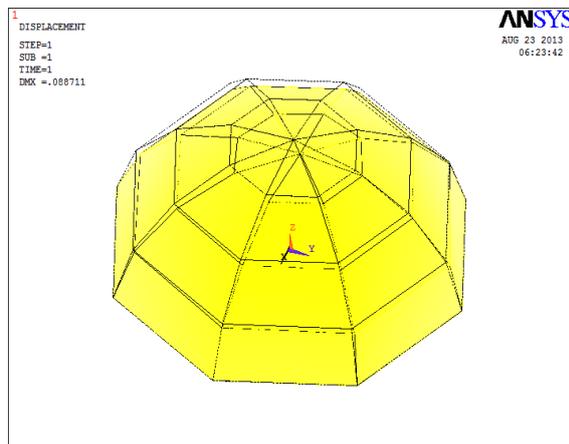


Fig. 14. Deflection due to horizontal force

Table 8 gives the Maximum deflection occurred in the dome structures using ANSYS. The Fig. 14 shows maximum deflection due to horizontal loading. The effect of shell element is more at higher  $H/D$  ratio. At higher ratios the deflection value reduced to above 20% of the initial value we obtained that means it has significance effect in the deflection and stability of the structure due to its membrane action. But when we

compare the deflection values at lower  $H/D$  ratio there is not much effect in the values.

#### 5.5 Buckling Load of Ribbed Dome

Failure of dome is generally due to buckling of the structure. It is a sudden failure occurs to the structure when it reaches a critical load, which is the maximum load which a member can support before it becomes unstable.

Table 9: Buckling Load of 20m Spanned Dome

| $H/D$ Raito | Buckling Load, $P_X 10^3$ [N] | % Buckling Load $P/P_u \times 100$ |
|-------------|-------------------------------|------------------------------------|
| 0.10        | 1150.4                        | 20.94                              |
| 0.15        | 1923.3                        | 35.00                              |
| 0.20        | 2775.5                        | 50.52                              |
| 0.25        | 3616.4                        | 65.82                              |
| 0.30        | 4346.9                        | 79.12                              |
| 0.35        | 4917.9                        | 89.51                              |
| 0.40        | 5290.1                        | 96.28                              |
| 0.45        | 5477.1                        | 99.69                              |
| 0.50        | $P_u = 5494.4$                | 100.00                             |

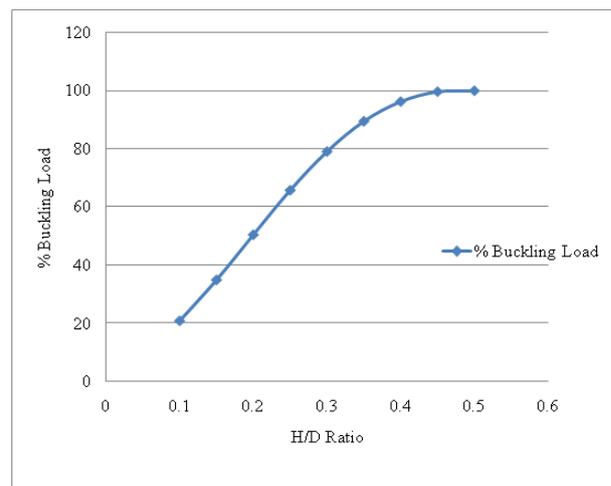


Fig. 15: % Buckling load Vs  $H/D$  Ratio for 20m Spanned Dome

From the table it is clear that maximum buckling load for a circular ribbed dome will be at a  $H/D$  ratio of 0.50, which is at semi spherical shape of circular dome buckling load will be maximum. The buckling load of dome is calculated for different rise to span ratios are calculated. From the graph we can propose suitable  $H/D$  ratio for the dome which has above 80% of the maximum buckling load. So we can choose the domes  $H/D$  ratio above 0.30

### III. CONCLUSION

Ribbed dome shows good performance against the vertical loads. Due to its structural symmetry and shape provide dome good performance

against vertical loading. For providing lateral stiffness to the dome structures, providing diagonal elements to the dome structures seems a good practice. We can reduce the section for rib if we provide diagonal members to the dome structure. Shell element shows significance effect in control of deflection due to horizontal loads. It is better to choose rise to span ratio in between 0.30 to 0.35 for ribbed dome, which can improve the performance of dome.

Future works should focus on determining the actual wind loads to have more realistic behaviour of the domes. There are various kinds of dome structures such lamella, Schwedler or geodesic domes. Comparing the efficiency of the domes with these domes can also be interesting. Another useful work can be the utilization of other types of sections or other materials. Including the effect of surface layer may change the behaviour of dome.

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